Candidate Dynamical Equation for the Serena Framework

Field valued in S^3 and Faddeev–Skyrme–type dynamics

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Abstract

We present a candidate dynamical equation compatible with the Serena Foundational Framework. The fundamental field takes values in the compact manifold $S^3 \subset \mathbb{C}^2$. From this field, an induced connection and a geometrically horizontal covariant derivative are defined, leading to an effective dynamics of Faddeev–Skyrme type. The goal of this work is to demonstrate the operational non-vacuity of the framework through a local, nonlinear, finite-order equation that is energetically stable. Effective radiative selection and full causal emergence are not addressed here and are deferred to future developments.

1 Context and scope

The Serena Foundational Framework has been introduced in previous works [1]. The present article has a deliberately limited scope: to exhibit an explicit dynamical equation compatible with that framework, establishing a concrete technical basis for subsequent mathematical and physical analyses.

This document constitutes a proof of concept. It does not aim to close the full phenomenological or foundational program associated with Serena Theory.

2 Fundamental field and geometric structure

Let

$$\psi(x) \in \mathbb{C}^2, \qquad \psi^\dagger \psi = 1,$$

be a field valued in the compact manifold S^3.

We define the induced connection

$$A_\mu := -i\,\psi^\dagger \partial_\mu \psi,$$

which is real as a direct consequence of the norm constraint.

The covariant derivative is defined as

$$D_\mu \psi := \partial_\mu \psi - i A_\mu \psi.$$

With these definitions, one identically has

$$\psi^\dagger D_\mu \psi = 0,$$

so that $D_\mu \psi$ is tangent to S^3 and horizontal with respect to the $U(1)$ fiber.

We introduce the tangential projector

$$\Pi_\psi := I - \psi \psi^\dagger.$$

3 Antisymmetric tensor and effective topology

We define the real antisymmetric tensor

$$S_{\mu\nu} := i\left((D_\mu\psi)^\dagger D_\nu\psi - (D_\nu\psi)^\dagger D_\mu\psi\right).$$

Due to the local gauge invariance induced by the connection, the effective physical dynamics is projected onto the quotient space

$$S^3/U(1) \simeq S^2.$$

Consequently, the model belongs to the class of Faddeev–Skyrme–type theories, whose stable bound states are characterized by the Hopf topological invariant.

4 Effective Lagrangian and stability

We consider the local Lagrangian

$$\mathcal{L} = 2c_1(D_\mu\psi)^\dagger D^\mu\psi - \kappa\, S_{\mu\nu}S^{\mu\nu}, \qquad c_1 > 0,\ \kappa > 0.$$

The quartic term guarantees the avoidance of trivial collapse in accordance with Derrick's theorem in three spatial dimensions, allowing the existence of extended, topologically nontrivial configurations.

5 Candidate dynamical equation

The associated Euler–Lagrange equation, restricted to the tangent space by projection, takes the form

$$\Pi_\psi\left[2c_1 D_\mu D^\mu\psi - 2\kappa D_\mu(S^{\mu\nu}D_\nu\psi)\right] = 0.$$

This equation is local, nonlinear, and of finite order. The constraint $\psi^\dagger\psi = 1$ is dynamically preserved.

6 Comments on metric and dissipation

6.1 Background metric

The covariant notation employed presupposes a fixed background metric (typically Minkowski), used as an auxiliary computational tool. No fundamental metric is postulated here.

In accordance with the Serena Framework, the effective causal structure must emerge from the analysis of perturbations and from the energetic content of the field, a question left for future work.

6.2 Radiative selection

The equation presented here derives from a conservative action principle. The mechanisms of effective radiative loss and dynamical selection of persistent states, which are central to the Serena Framework, are not explicitly incorporated in this version.

The equation should therefore be understood as a dynamical base upon which such mechanisms may be analyzed asymptotically or effectively.

7 Conclusion

We have presented a candidate dynamical equation coherent with the Serena Framework, with a consistent geometric structure and an effective topology of Faddeev–Skyrme type. The result establishes a non-vacuous and well-defined mathematical basis for exploring topological persistence, causal emergence, and the discreteness conjecture in future developments.

References

[1] J. E. P. Argibay, *Serena Theory as a Pre-Dynamical Foundational Framework*, Zenodo (2025), https://doi.org/10.5281/zenodo.18059332doi:10.5281/zenodo.18059332.