Conceptual foundations and motivation of the axioms of Serena theory

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Abstract

This document presents the conceptual framework underlying Serena theory and describes the motivation of the axioms employed in its current formulation. The general postulates establish a broad physical idea: a single field of the medium, endowed with internal structure and its own symmetry, from which stable configurations and interactions emerge. From these postulates, a concrete realization is adopted based on a field valued in S^3, an internal $U(1)$ symmetry, and a strictly local dynamics. This choice is understood as the minimal and coherent option that allows the basic mathematical structure to be developed and enables progress on the technical aspects of the theory.

1 Introduction

Serena theory is based on a central idea: the possibility of describing physical phenomenology as the manifestation of a continuous medium endowed with internal degrees of freedom and subject to local variations that produce stable configurations. To clarify this conceptual structure, it is useful to distinguish between:

1. General postulates: broad principles that guide the theory.
2. Concrete axioms: a specific realization based on those postulates.
3. Dynamical consequences: the effective Lagrangian that follows from the axioms.

Separating these levels prevents the current axioms from being interpreted as the final form of the theory and facilitates future extensions without compromising the foundations.

2 General postulates

The postulates express the essential physical idea without fixing a definitive mathematical formulation:

1. **Existence of a single field of the medium.** A continuous field is postulated, permeating space, whose dynamics allow the appearance of stable configurations and observable phenomena.
2. **Internal structure with its own symmetry.** The field possesses internal degrees of freedom organized by a symmetry, whose concrete form is not specified at this stage.
3. **Emergent phenomenology from local variations.** Matter, interactions, and geometry emerge from configurations of the field itself, without introducing additional entities.

3 Axioms of the current formulation of Serena

To develop a concrete formulation, three axioms compatible with the previous postulates are adopted. They represent the minimal choice that allows the construction of a manageable Lagrangian and a coherent geometric structure.

1. **Fundamental field.** A field ψ valued in S^3, subject to the constraint

$$\psi^\dagger \psi = 1.$$

   All relevant objects are derived from this field.

2. **Internal symmetry.** Invariance under local phase transformations

$$\psi \to e^{i\alpha(x)}\psi.$$

3. **Locality.** The action depends only on the field ψ and a finite number of its first derivatives.

   No additional, unmotivated structures are introduced.

These axioms represent the simplest and most intuitive choice arising from the general postulates and allow the mathematical structure of Serena theory to be constructed in a natural way.

4 Motivation of the axioms

4.1 Justification of the internal space S^3

The choice of S^3 is motivated by the fact that it provides:

- nontrivial topology, necessary for the existence of knots,
- a compact structure with good mathematical control,
- the natural appearance of the Hopf map and the director field n,
- the ability to produce stable configurations without adding external terms.

It is the simplest option that satisfies the requirements derived from the postulates.

4.2 Justification of the $U(1)$ symmetry

The $U(1)$ symmetry is the simplest one that allows:

- the definition of an emergent connection A_μ,
- the construction of a curvature tensor $F_{\mu\nu}$,
- the association of a conserved charge via the Noether current,
- the generation of gauge-type interactions without unnecessarily complicating the theory.

It is a contained and adequate choice for exploring the basic behavior of the model.

4.3 Justification of the locality principle

Adopting locality allows one to:

- derive clear equations of motion,
- avoid artificial or unmotivated terms,
- ensure a minimal and controllable Lagrangian,
- maintain continuity with established principles in theoretical physics.

Although locality may be revisited in later versions, it is the most reasonable choice at this stage.

5 Conclusion

The axioms adopted in the current formulation of Serena theory constitute the minimal and coherent realization of the general postulates. Separating the theory into postulates, axioms, and dynamical consequences improves conceptual clarity and allows future extensions or refinements to be integrated without altering the core of the proposal.

The present formulation should be regarded as a solid starting point for the mathematical and numerical development of the theory, including the explicit construction of knot-type solutions and the detailed study of their emergent structures.