1 Explicit static candidate for the $Q = 1$ sector

1.1 Objective

We define an explicit trial configuration

$$\Psi_1^{\text{trial}} \in X_1$$

satisfying:

- it belongs unambiguously to the topological sector $Q = 1$,
- it is a connected (non-fragmented) configuration,
- it serves as a reference energy E_1 in fragmentation criteria.

1.2 Structure of the model

We consider a field

$$\psi : \mathbb{R}^3 \to S^3 \subset \mathbb{C}^2, \qquad \psi^\dagger \psi = 1, \qquad \psi(x) \to \psi_\infty \quad \text{as } |x| \to \infty.$$

The vacuum condition compactifies the domain:

$$\mathbb{R}^3 \cup \{\infty\} \simeq S^3.$$

1.3 Topological construction

Let $(z_1, z_2) \in \mathbb{C}^2$ be homogeneous coordinates on S^3:

$$|z_1|^2 + |z_2|^2 = 1.$$

Define the rational map

$$W_1(z_1, z_2) := \frac{z_1}{z_2}.$$

This induces a map

$$W_1 : S^3 \longrightarrow \mathbb{CP}^1 \simeq S^2$$

representing the fundamental Hopf class.

From W_1 we construct a director field

$$\mathbf{n}_1 \in S^2$$

and a compatible lifted configuration

$$\Psi_1^{\text{trial}} := \psi_{W_1} \in X_1.$$

Remark 1. *The configuration Ψ_1^{trial} carries exactly one unit of topological charge:*

$$Q(\Psi_1^{\text{trial}}) = 1.$$

1.4 Bound character

The configuration Ψ_1^{trial} is a single connected object:

- it contains no separation parameters,
- it is not a superposition of lower-charge configurations,
- it does not admit asymptotic decomposition.

It therefore represents the elementary static object of the spectrum.

1.5 Internal scale and reference energy

Introduce the dilation family

$$\psi_{1,R}^{\text{trial}}(x) := \psi_1^{\text{trial}}(x/R), \qquad R > 0.$$

For a functional containing quadratic and quartic terms, the energy reads

$$E_1^{\text{trial}}(R) = A_1 R + \frac{B_1}{R}, \qquad A_1 > 0,\ B_1 > 0.$$

The minimum is explicit:

$$R_{1,\star} = \sqrt{\frac{B_1}{A_1}}, \qquad E_{1,\star}^{\text{trial}} = 2\sqrt{A_1 B_1}.$$

1.6 Role in the Serena framework

The configuration Ψ_1^{trial}:

1. fixes the base energy scale,
2. defines the fragmentation threshold,
3. serves as a building block for higher-Q sectors.